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Instability of Superposed Walters' B′ fluid in a Porous Medium

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Abstract

This paper deals with the study of hydrodynamics instability of superposed Walters' B' fluid overlying viscous fluids in a porous medium. The stability or instability of the concerned system has been studied under the corresponding configuration. The effects of medium permeability, medium porosity and the ratio of fluid densities on the flow have been analyzed within the purview of the boundary conditions adhering to the super position of visco-elastic fluids over viscous fluids. It is observed that the visco-elasticity of the fluid makes the system unstable even for potentially stable configuration Medium Permeability has influenced the stability of the system to a great extent. The system is stable or unstable according to the kinematic visco-elasticity being less than or greater than a quantity depending upon medium permeability, medium porosity and ratio of fluid densities.

Keywords:

Hydrodynamics;
Instability;
Walters' B′ fluid;
Porous medium.

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IMPORTANCE OF THE STUDY

The stability derived from the character of the equilibrium of an incompressible heavy fluid of variable density is termed as Rayleigh-Taylor instability. The Rayleigh-Taylor instability problems arise in the oceanography, limnology and engineering. The problem of Rayleigh-Taylor instability in the fluids in a porous medium is of great importance in geophysics, soil science, ground water, hydrology and astrophysics.

BACKGROUND

The instability of a plane interface between two incompressible viscous fluids of different densities, when the lighter one is accelerated into the heavier one, has been discussed by Chandrasekhar[1]. The influence of viscosity on the stability of the plane interface separating two incompressible superposed conducting fluids of uniform densities, when the whole system is immersed in a uniform horizontal magnetic field, has been studied by Bhatia[2]. He has carried out the stability analysis for two highly viscous fluids of equal kinematic viscosity and different uniform densities. Sharma and Sharma[3] have studied the Rayleigh-Taylor instability of two viscoelastic (Oldroyd's) superposed fluids of uniform densities.

There are many viscoelastic fluids that cannot be characterized by Maxwell or Oldroyd's constitutive relations. One such class of elastico-viscous fluids is Walters (model B') fluid. With growing importance of non-Newtonian fluids in chemical engineering, modern technology and industry, the investigations on such fluids are desirable. In recent years, the investigation of flow of fluids through porous media has become an important topic due to recovery of crude oil from the pores of reservoir rocks. The flow through porous

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media is of considerable interest for petroleum engineers and for geophysical fluid dynamicists. When a fluid flows through a porous medium, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous and viscoelastic terms in the equation of motion are replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t} \right) q \right]$, where *m* and *m¢* are the viscosity and viscoelasticity of the Walters' (model B¢) fluid, k_1 is the medium permeability and *q* is the Darcian (filter) velocity of the fluid.

Most often in geophysical situations, the fluid is not pure but usually permeated with suspended particles. The effect of suspended particles on the stability of stratified fluids in porous medium might be of industrial and chemical engineering importance. Further, motivation for this study is the fact that knowledge concerning fluid particle mixtures is not commensurate with their industrial and scientific importance. Sharma and Rajput[4] considered the effect of suspended particles on Rayleigh-Taylor instability.

In stellar interiors and atmospheres, the magnetic field may be (and quite often is) variable (and non-uniform) and may altogether alter the nature of the instability. Sharma and Sunil[5] have studied the Rayleigh-Taylor instability of partially ionized plasma in a porous medium in presence of a variable magnetic field. Sharma and Kumar[6] have studied the stability of the plane interface separating two visco-elastic (Walters' B') superposed fluids of uniform densities. Kumar[7] studied Rayleigh-Taylor instability of Rivilin-Ericksen elastico viscous porous medium. Sharma *et al*.[8] have considered the Rayleigh-Taylor instability of Walter' B' fluid through porous medium. Kumar *et al*.[9] studied the stability of two superposed viscoelastic fluid particle mixtures. Prakash and Aggarwal[10] have analysed the stability of superposed viscoelastic (Walters' B') fluids in porous medium. Copious literature on the criterion for the on set of convection due to non-uniform temperature gradient either in a fluid layer (Currie[11], Nield[12], Zangrando and Bertram[13]) or in a fluid saturated porous medium (Nield[14], Rudraiah[15] *et al.*) is available. However, the effect of non-uniform temperature gradient on the on set of convection in a composite layer has not been given much attention inspite of its applications in practical problems like thermal insulation, grain storage, heat exchangers, oil-extraction and catalytic reactors. In the present paper, instability of superposed Walters' B' fluid over viscous fluid in a porous medium has been studied with the thermal energy transmission.

Formulation of the Problem and Perturbation Equations

Consider a static state in which an incompressible Walters' B¢ fluid-particle layer of variable density is arranged in horizontal strata in a porous medium and the pressure *p*, density *r*, viscosity *m*, suspended particles number density *N* are functions of the vertical coordinate *z* only. The character of the equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then follows its further evolution.

Let *r*, *p* and *q* (*u*, *v*, *w*) denote respectively the density, the pressure and the filter velocity of the pure fluid : q_d (*x*, *t*) and r_d (*x*, *t*) denote the velocity and density of the suspended particles, respectively. $F = 6\pi\mu\nu r$, where *r* is the particle radius, *F* is the Stokes' drag, $q_d = (l, r, s)$, $x = (x, y, z)$ and $\hat{k}$ = (0,0,1). Let $\varepsilon, k_1, \mu, \mu'$ and *g* stand for medium porosity, medium permeability, coefficient of viscosity of fluid, viscoelasticity of fluid and acceleration due to gravity respectively. Then the equations of motion and continuity for the Water' B' viscoleastic fluid permeated with suspended particles through porous medium (Sharma and Kumar[6] and Yih[10]) are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q.\nabla) q \right] = -\nabla p - \rho g \hat{k} + \frac{F \rho_d}{m\varepsilon}(q_d - q) - \frac{1}{k_1}\left(\mu - \mu' \frac{\partial}{\partial t} \right) q, \qquad (1)$$

$$\nabla . q = 0 \qquad (2)$$

In writing equation (1), we have assumed uniform particle size, spherical shape and small relative velocities between the two phases. Then the net effect of the suspended particles on the fluid through porous medium is equivalent to an extra body force term per unit volume $\frac{F \rho_d}{m\varepsilon}(q_d - q)$. Since the force exerted by the fluid on the particles is equal and opposite that exerted by the particles on the fluid, there must be an extra force term equal in magnitude but opposite in sign in the equations of motion of the particles. The distances between particles are assumed so large compared with their diameter that inter particle reactions need not be accounted for. The effects of pressure, gravity and Darcian force on the suspended particles, assumed large distances apart, are negligibly small and therefore ignored. If r_d is the mass of particles per unit volume, then the equations of motion, energy and continuity for the particles, under the above assumptions are

$$\rho_d\left[\frac{\partial q_d}{\partial t}+\frac{1}{\varepsilon}(q_d.\nabla)q_d\right]=F\frac{\rho_d}{m}(q-q_d),$$

(3)

$$(q_d.\nabla)T=K\nabla^2 T,$$

(4)

$$\varepsilon\frac{\partial\rho_d}{\partial t}+\nabla.(\rho_d q_d)=0.$$

(5)

Since the density of a fluid particle moving with the fluid remains unchanged, we have

$$\varepsilon\frac{\partial\rho}{\partial t}+(q.\nabla)\partial=0.$$

(6)

Let $\delta\rho,\delta p, q(u,v,w)$ and $q_d(l,r,s)$ denote respectively the perturbations in density r, pressure *p*, fluid velocity (0,0,0) and particle velocity (0,0,0), then the linearized perturbation equations of the Walters' B¢ fluid-particle layer are

$$\frac{\rho\partial q}{\varepsilon\partial t}=\nabla\delta p-g\,\delta\rho\,\hat{k}+\frac{F\rho_d}{m\varepsilon}(q_d-q)-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)q,$$

(7)

$$\nabla.q=0.$$

(8)

$$\left(\frac{m}{F}\frac{\partial}{\partial t}+1\right)q_d=q,$$

(9)

$$\varepsilon\frac{\partial}{\partial t}\delta p=-w(D\rho),$$

(10)

$$\frac{\partial M}{\partial t}+\nabla.q_d=0.$$

(11)

where $M=\frac{\in\rho_d}{\rho_0}$ and ρ_0 , ρ_d stand for the initial uniform fluid density and density of suspended particles respectively and *D* stands for $\frac{d}{dz}$.

$$\rho_d=\rho_0\left[1-\alpha(T_d-T_0)\right]$$

Analyzing the disturbances into normal modes, we seek solutions whose dependence on *x*, *y* and *t* is given by

$$\exp\,(ik_x x+ik_y y+nt)$$

(12)

where *n* is, in general a complex constant, k_x, k_y are wave numbers along *x* and *y* directions and $k^2=k_x^2+k_y^2$.

For perturbations of the form (12), equations (7) – (10), after eliminating q_d, yield

$$\frac{\rho}{\varepsilon} nu = -ik_x \delta p - \frac{1}{k_1}(\mu - \mu' n)u - \frac{\rho_d}{\varepsilon(\tau n+1)} nu,$$
(13)

$$\frac{\rho}{\varepsilon} nv = -ik_y \delta p - \frac{1}{k_1}(\mu - \mu' n)v - \frac{\rho_d}{\varepsilon(\tau n+1)} nv,$$
(14)

$$\frac{\rho}{\varepsilon} nw = -D\delta p - \frac{1}{k_1}(\mu - \mu' n)w - \frac{\rho_d}{\varepsilon(\tau n+1)} nw - g\delta\rho,$$
(15)

$$ik_x u + ik_y v + Dw = 0 ,$$
(16)

$$\varepsilon n \delta p = -(D\rho) w,$$
(17)

where $\tau = \frac{m}{F}$. Eliminating δp between equation (13) – (15) and using equations (16) and (17), we obtain

$$\frac{n}{\varepsilon} = \left[D(\rho Dw) - k^2 \rho w \right] + \frac{1}{k_1}\left[D\{(\mu - \mu' n) Dw\} - k^2 (\mu - \mu' n) w \right]$$
$$+ \frac{n}{\varepsilon(\tau n+1)}\left[D\{\rho_d Dw\} - k^2 \rho_d Dw \right] - \frac{gk^2}{\varepsilon n}(D\rho) w .$$
(18)

Walters' B' Viscoelastic Fluid overlying Viscous Fluid separated by Horizontal Boundary

Consider the Walters' B′ viscoelastic fluid of density ρ_2, viscosity μ_2, visco-elasticity μ_2' suspended particle number density ρ_2' overlying the viscous fluid of density ρ_1, viscosity μ_1 and suspended particle number density ρ_1' and separated by a horizontal boundary at $z = 0$. The subscript 1 and 2 distinguish the lower and upper fluids respectively (Fig.1). Obviously we choose a co-ordinate system with x-axis parallel to the channel and z-axis vertically upwards with the origin at the interface. The fluid occupies the region $0 \le z \le h$ and the porous layer the region $-h \le z \le 0$. The whole system is bounded between the horizontal boundaries which are maintained at constant heat flux and both rigid. The upper and lower boundaries are maintained at T_u and T_L respectively and τ_0 is the temperature at the interface, where $T_L > T_u$. Then in each region of constant r, constant μ, constant μ' and constant ρ_d, equation (18) reduces to

$$(D^2 - k^2) w = 0$$
(19)

which is similar to equation (45) of Chandrasekhar[1] in case of superposed inviscid fluids.
The general solution to equation (19) is

$$w = Ae^{+kz} + Be^{-kz}$$
(20)

where A and B are arbitrary constants.

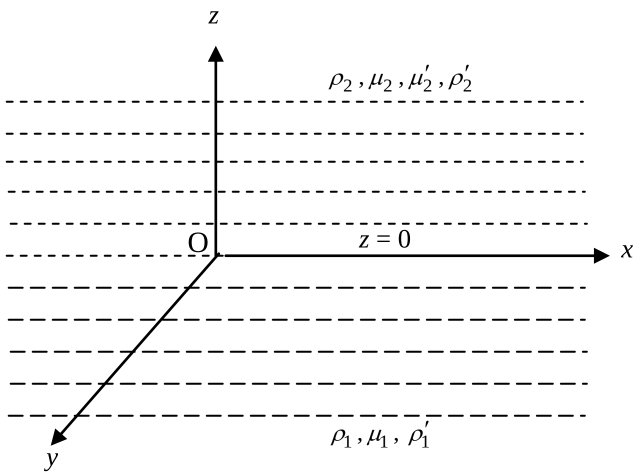


Fig. 1 : Geometrical configuration

The boundary conditions to be satisfied in the present problem are –

1. The velocity $w \to 0$ when $z \to +\infty$ (for the upper fluid) and $z \to -\infty$ (for the lower fluid)
2. $w(z)$ is continuous at $z = 0$
3. The jump condition at the interface $z = 0$ between the fluids. This jump condition is obtained by integrating equation (18) across the interface at $z = 0$ and is

$$\frac{n}{\varepsilon}(\rho_2 Dw_2 - p_1 Dw_1)_{z=0} + \frac{1}{k_1}\left[\left(\mu_2 - \mu_2' n\right) Dw_2 - \mu_1 Dw_1\right]_{z=0}$$
$$+ \frac{mn}{\varepsilon(\tau n + 1)}(N_2 Dw_2 - N_1 Dw_1)_{z=0} = -\frac{gk^2}{\varepsilon n}(\rho_2 - \rho_1) w_0$$

(21)

where w_1 is the common value of w at $z = 0$.

Applying the boundary conditions (1) and (2), we can write from equation (20).

$$w_1 = Ae^{kz}, \ (z < 0)$$

(22)

$$w_2 = Ae^{-kz}, \ (z < 0)$$

(23)

where the same constant A has been chosen to ensure the continuity of w at $z = 0$. Applying the condition (21) to the solution (22) and (23), we obtain

$$\tau\left(1 - \frac{\varepsilon}{k_1} v' \alpha_2\right) n^3 + \left[1 - \frac{\varepsilon}{k_1} v' \alpha_2 + \frac{\varepsilon\tau}{k_1} v + \frac{m(\rho_1' + \rho_2')}{(\rho_1 + \rho_2)}\right] n^2$$
$$+ \left[\frac{\varepsilon}{k_1} v - gk\tau\,(\alpha_2 - \alpha_1)\right] n - gk(\alpha_2 - \alpha_1) = 0$$

(24)

where

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}, \quad v_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}, \quad v_{1,2}' = \frac{\mu_{1,2}'}{\rho_{1,2}}.$$

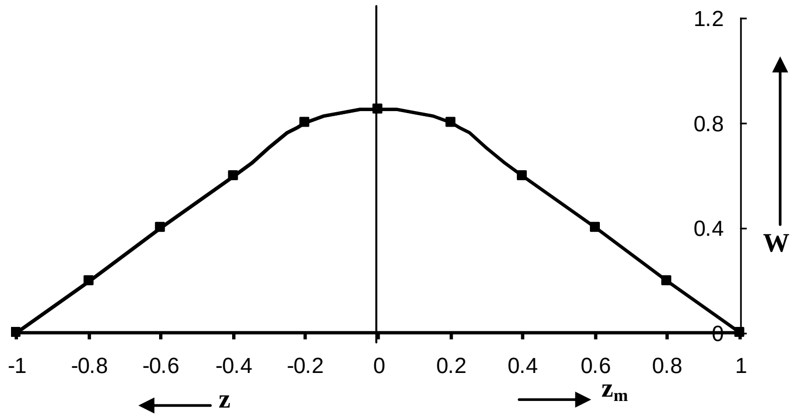
In deriving equation (24) we have made the assumption that the kinematic viscosities and kinematic viscoelasticities of the two fluids are equal, i.e. $v_1 = v_2 = v$ and $v_1' = v_2' = v'$. However, his simplifying assumption does not obscure any of the essential features of the problem.

Case I – Stable case $(\rho_2 < \rho_1)$:

For the potentially stable arrangement $(\rho_2 < \rho_1)$ and $\nu' < \frac{k_1}{\varepsilon}\frac{(\rho_1+\rho_2)}{\rho_2}$, all the coefficients of equation (24) are positive. Therefore, all the three roots of equation (24) are either real and negative or there is one real negative root and the other two are complex conjugates with negative real parts. The system is, thus, stable in each case.

However, for the potentially stable arrangement $(\rho_2 < \rho_1)$ and $\nu' < \frac{k_1}{\varepsilon}\frac{(\rho_1+\rho_2)}{\rho_2}$, the coefficient of n^3 in equation (24) is negative. Equation (24), therefore, allows one change of sign and so has one positive root. The system is therefore unstable.

We thus conclude that for superposed viscoelastic (Walters' B′)-viscous fluids permeated with suspended particles in porous medium and for the potentially stable arrangement, the system is stable or unstable according to the kinematic viscoelasticity being less than or greater than a quantity depending upon the medium permeability, medium porosity and ratio of fluid densities. This is in contrast to the stability of Newtonian superposed fluids permeated with suspended particles in porous medium, where the system is always stable for the stable configuration (Prakash and Manchanda[11]). The velocity profile for rigid-rigid boundaries as a function of *z* is given in the Fig.2. It is revealed from this figure that the velocity first rises with *z* and then falls to attend zero value at *h* = 1.



Case II – Unstable case $(\rho_2 > \rho_1)$:

For the potentially unstable arrangement $(\rho_2 > \rho_1)$, the constant term in equation (24) is negative. Equation (24) has a change of sign and hence allows one positive root. The occurrence of positive root implies instability of the system. Thus for the potentially unstable case, the system is unstable for superposed viscoelastic (Walters' B′) – viscous fluids permeated with suspended particles in porous medium.

Heat Transfer :

In the porous medium, we have

$$-\frac{h}{(T_L-T_0)}\frac{dT_d}{dz} = g_1(z) \ , \tag{25}$$

$$P_d = P_0 - \rho_0\, gz - \left(\frac{\rho\alpha g}{2h}\right)\left(T_L - T_0\right)z^2 \ , \tag{26}$$

where $g_1(z)$ is a non-dimensional temperature gradient in the porous medium.

The interface temperature and pressure are given by

$$T_0 = \frac{-K'hT_u + K\,d\,T_L}{-K'\,h + K\,h}$$

(27) $$P_0 = P_a + \rho_0\, g\, h + \left[\left(\rho_0\, \alpha\, g\right)/\, 2h\right]\left(T_0 - T_u\right),$$

(28)
where P_a is the pressure at $z = h$. After employing the usual process of linearization, the perturbation-equation for temperature is given by

$$g_1(z)(T_L - T_0)\,(-h)^{-1}\, W + K\nabla^2\,\theta = 0\,,$$

(29)
with the boundary conditions

$$\left.\begin{aligned} &\text{at } z = h\;,\;\theta = 0\;,\;\frac{\partial\theta}{\partial z} = 0 \\ &\text{at } z = 0\;,\;\theta = \theta'\;,\;K'\frac{\partial\theta}{\partial z} = K\frac{\partial\theta}{\partial z} \\ &at\;\; z = -h\;,\;\frac{\partial\theta'}{\partial z} = 0 \end{aligned}\right\}$$

(30)
We use the non-dimensional quantities as

$$\left.\begin{aligned} &\vec{q} = \frac{k}{h}\vec{q}'\;\;,\;\theta = (T_0 - T_u)\;\theta'\;, \\ &(x, y, z) = h(x', y', z')\;,\;\; P = \frac{\mu K}{h^2}\,\rho', \text{Prandtl number} \\ &R' = \alpha g\,(T_0 - T_u)\,h^3 / \nu k', \text{ Rayleigh number} \\ &R = \alpha g\,(T_L - T_0)(-h)\;k/\nu k, \text{ Rayleigh Darcy number} \\ &\sigma = \frac{(-h)}{\sqrt{k'}}\;,\text{ Porousity parameter} \end{aligned}\right\} \qquad (31)$$

The boundary conditions involving θ' and θ will give rise to the solvability condition

$$\int_0^1 f(z) W_1\, dz + \hat{h}^2\int_0^1 g_1(z_m)\; W_{m1} dz_m = \left(\hat{T} + \hat{h}^2\right), \qquad (32)$$

This forms a stability equation. The eigen values R_0 using the relation

$$W_1 = \frac{R_0\hat{T}}{24}\left(z^4 + c_3 z^3 + c_2 z^2 + c_1 z + c_0\right),$$

(33)

Both boundaries rigid :

When both the boundaries are rigid, the critical Rayleigh number is obtained under the adiabatic condition. So the corresponding boundary conditions at the upper and lower boundaries are

$$\left.\begin{aligned} &W_1(1) = DW_1(1) = D\theta_1(1) = 0 \\ &W_{m1}(0) = D_m W_{m1}(0) = D_m\theta_{m1}(0) = 0 \end{aligned}\right\} \qquad (34)$$

The interfacial boundary conditions remain the same in all the boundary combinations. When the fluid layer is heated from below at a constant rate, $f\ (z)$ and $g_1(z_m)$ are not only positive but also decreases monotonically. A suitable temperature profile can be obtained for optimizing R_0. In this context, the basic temperature profiles predicted by Sheela[16] are presented in Fig.3.

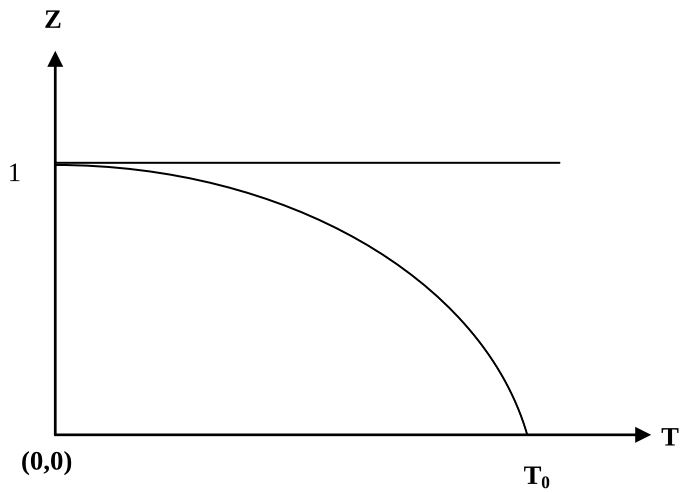


Fig. 3 : Temperature profile

Conclusion

The instability of superposed viscoelastic (Walters' B′) fluid overlying viscous fluids has been studied to include the effect of suspended particles. The system is found to be stable or unstable if the kinematic viscoelasticity is less than or greater than a quantity depending upon the medium permeability, medium porosity and ratio of fluid densities, for potentially stable arrangement, whereas the system is unstable for the potentially unstable arrangement. This is in contrast to the stability of two superposed Newtonian fluids in the presence of suspended particles in porous medium where the system is always stable for the stable configuration. The viscoelasticity, thus, makes the system unstable even for potentially stable configuration. The temperature profile resembles a parabola as shown in Fig.3. The temperature becomes T_0 when z_0.

References

1. Chandrasekhar, S., "*Hydrodynamic and Hydromagnetic Stability"*, Dover Publication: New York.,1981.
2. Bhatia, P.K., " *Nuovo Cimento"*, Vol.19B, PP. 161-168, 1974.
3. Sharma, R.C. & Sharma, K.C., " *Acta Physica Hungarica*", Vol**.** 45, PP. 213-220,1978.
4. Sharma, R.C. & Rajput, A. *Czech., J. Phys.*, Vol**.** 42**,** PP. 919-926, 1992.
5. Sharma, R.C. & Sunil, *Zeitschrift fur Naturforschung, Vol.* 47a, PP. 1227-1231, 1992.
6. Sharma, R.C. & Kumar, P., *Czech. J. Phys.* Vol. 47, PP. 197-204, 1997.
7. Kumar, P., *Indian J. Pure Appl. Math*., Vol. 31, PP**.** 533-539, 2000.
8. Sharma, R.C., Kumar, P. & Sharma, S., *Int. J. Appl. Mech. Enging.*, Vol.**7**, PP. 433-444, 2002.
9. Kumar, P., Mohan, H. & Singh, G.J., *ZAMM*, Vol.86, PP. 72-77, 2005.
10. Prakash, K. & Manchanda, S., *J. Math. Phy. Sci.*, Vol.28, PP**.** 75-87, 1994.
11. Currie, I.G., *J. Fluid Mech.*, Vol.29, PP**.** 337, 1967.
12. Nield, D.A., *Convective flow in Porous media*, Ed. By R.A. Wooding and I. White, IPS, Newzealand, 1984.
13. Zangrando, F. and Bertram, A.L., *J. Fluid Mech*.,Vol. 151, PP. 55, 1985.
14. Nield, D.A., *J. Fluid Mech*., Vol.81, PP. 513, 1977.
15. Rudraih, N., Veerappa, B. and Balachandra Rao, S., *Int. J. Heat and Mass Transfer*, Vol.25, PP. 1147, 1982.
16. Sheela, R., *Stability of flow through and past porous media*, Ph.D. Thesis, Bangalore University, Bangalore, 1990.